Types of Errors Committed by Senior Secondary School Students in Graphical Solution to Inequality Problems

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ABSTRACT

The study was set out to identify the types of errors the senior secondary school class two students commit while solving inequality problems using graphical method. The design of the study was descriptive survey. Out of the total   population of 1782 senior secondary school class two students from public schools in Aguata Education Zone in Anambra State, the study used 92 students from six schools selected through a multiple random sampling method. All the students in each of the six selected schools were administered with the instruments for data collection in their intact classes with the help of their regular class teachers. The instruments for data collection were a five Essay-type structured questions on inequality in both one and two variables and an 11-item structured questionnaire. The reliability coefficient of the essayquestions was 0.72 established through inter – rater method and that of the questionnaire was 0.82. The data collected was analyzed using the simple frequencies and percentage score and chi-square. Three research questions and one null hypothesis guided the study. The findings showed that most of the students used in the study commit such errors as errors in representing inequalities when the coefficient of y variable is negative, and errorsin representinginequality on the number line and on the Cartesian plane. It was also found out that one of the strategies to be used by the teacher to minimize the errors is that teachers should administer readiness test to the students to identify ability to plot ordered pairs and read graphs before introducing the topic. The researchers therefore recommended among others that skills relating to scales and plotting of ordered pairs (x, y) on the Cartesian plane should be clearly and adequately taught and practiced with the students.
Keywords: error, inequality, graphical method, problems.

INTRODUCTION

There has been continuous emphasis and studies on how to ensure effective teaching and learning of Mathematics at all levels of schooling in Nigeria [1,2]. Some of these studies laid great emphasis on the primary and secondary levels which form the basis for further studies in the discipline. [3,4] argued that children are capable of far deeper Mathematical thinking than had previously been thought and expressed that children think differently than adults do. In the light of the above belief, the teachers of mathematics especially those at the primary and secondary levels, must be individuals who possess beliefs and knowledge to engage in classroom practices that relate to four issues of Mathematics: content, pedagogy, child development and student thinking, in an integrated way [5,6]. In line with this thought, [7] in [8] suggested encouraging system with change in the way mathematics is conceptualized, learned and taught in schools [9]. The systemic change can be ensured by professional development, curriculum materials and assessment.Most research works in mathematics education investigate some factors both in contents and pedagogythat contribute to successes and poor performance of students in mathematics at both primary and secondary school levels [10]. This is becauseMathematics competence at these levels is a criticaldeterminant of the Post-Secondary educational and career options available to young people. The place of

Mathematics in the life of any nation cannot be overemphasized because it is linked with the place of development in that nation. According to [11], science, Technology and Mathematics Education (STME) has been widely acclaimed to be the index of measuring any nation's socio economic and geo-political development. Among science and Technology courses, according tothe National Policy on Education [12], Mathematics is one of the core subjects to be offered by all students till the tertiary levels of education. Its use as a language for clear and concise expression of the contents, principles, laws and theories in other related disciplines is clear. The application of Mathematical ideas, principles and rules in almost every aspect of human life cannot be compromised at the altar of any type of instructional activities in the schools. A number of mathematical concepts have wide applications in industries, business transactions, population studies, construction works, banking activities; agricultural activities etc. One of such concepts is inequality which at a higher level deals with linear mathematical programming. [13] defined inequality as a mathematical statement that consisted of two expressions that are not equal. Linear mathematical programming on the hand is a branch of mathematics that deals with the problem of finding maximum or minimum values of a linear function under limiting conditions, when the variables under consideration are connected by a number of linear inequalities [14].The [15] observed that graphing inequality on either a number line or in the coordinate plane (x and y axes) forms visual representation of several forms of inequalities. The graph produces a region on the coordinate plane with boundary line and every point in that region is a solution of the inequality.

In spite of the importance of this concept and other areas of Mathematics, students' general performance has not been encouraging.WAEC chief examiners report [16] observed that students performance in Senior School Certificate Examination (SSCE) in Mathematics have been poor over the years. WAEC Chief Examiners reported that one of the areas where students committed errors was in inequality problems involving graphs.The Oxford Advanced Learners Dictionary defined error as the state, quality or condition of being wrong. In an attempt to find solution to a mathematical task, the candidate may commit one error or another some of these can be wrong application of rule, premature approximation, wrong lifting of data or information, omission of essential steps, wrong definition, misconception of idea, wrong measurement, omission of units etc.[4] in a study on difficulties in learning inequalities in students of the first year of pre-university education in Spain, revealed that most students could not draw conclusions from what they did, lacked mastery of elementary algebra, could not apply the correct inequality signs and showed poor knowledge of the inequality interval. [6] in a study on error Analysis of solving linear inequalities among senior secondary school students observed that the students have difficulty in solving inequalities especially in using and applying the inequality rules such as, changing of inequality sign when multiplied or divided by a negative number. The author also observed that pre-service teachers incorrectly represent the solutions of inequalities on the number line. The result is in line with the finding of [8] who identified a number of errors committed by candidates in their attempt to solve inequality problems involving one and two variables. The author observed that candidates generally experience difficulties in the collection of like terms especially in the way they collect and drop the signs carelessly.

[11] observed that common errors students make, learning to solve equations is the interpretation of the way multiplication is represented.Inequality problems are not greatly different from simple linear equations except when negative signs are involved. [2] found out from their analysis of misconception held by mathematics students in algebraic

equations, that students had difficulties in separation and transformation of algebraic terms, balancing and knowledge of algebraic structures. They also found out that students commit errors such as invalid procedure for simplification, wrong algorithms, and invalid distribution when solving algebraic equations.

Engaging students in Mathematical activities meaningfully under the watchful eyes of committed, knowledgeable and passionately focused and pedagogically balanced mathematics teachers will provide opportunity to discover students' areas of strengths and weakness in a given mathematical task. Under this type of classroom practice, student's errors will provide opportunity for better instructional practices through immediate and continuous formative assessment. It is in this light that the researchers set out to identify the types of errors senior secondary school students commit while solving graphical problems involving inequality with a view toproffer instructional strategies to reduce the occurrence of such errors.

The investigation was guided by the constructive theory of Dewey the philosophical founder of the theory in [4]. The theory is anchored on the idea that learners must be actively involved by constructing knowledge and skills as they reflect on life experiences. Learners grow in knowledge as they construct their own understanding of the world around them. They therefore adjust their mental models in order to accommodate new experiences.The teacher should provide opportunity for the learners to show this competency through active participation and responses in class Mathematical tasks.

STATEMENT OF THE PROBLEM

Students' performance in mathematics in schools has not been encouraging. The effect of the poor performance of candidates in the subject becomes more frustrating and worrisome when one considers the thousands of secondary school leavers who are denied admission into tertiary institutions because of failures in mathematics. Many candidates are also in courses they have no flare for due to inability to make credit pass in Senior School Certificate Examinations in Mathematics. WAEC chief examiners' reports for the period 2006 to 2016, observed that annual results in west African senior school certificate examination in Mathematics released by the council, have remained poor. Many of the candidatesfailed to make the minimum credit requirement for entry into tertiary institution.This situation if not checked, will continue to affect the quality of school graduates and mismatch of individuals and career prospects in general.The concept of inequality is supposed to be an interesting aspect of secondary school mathematics because of its applicability in the three domains of educational objectives which are cognitive, affective and psychomotor. But it's disheartening to know thatmost of the candidates who took mathematics examination, commit a number of errors in attempting problems involving graphical solution to inequality [4]. Most of the studies in Nigeria investigated the difficulties students encounter using algorithm to solve linear inequalities in one and two variables. This situation prompted the need for this study in Aguata Education Zone of Anambra State among Secondary School Students. This study therefore set out to fill this gap by investigating into the types of errors senior secondary school students commit as they solve linear inequality problems in one and two variables using graphical method.

PURPOSE OF THE STUDY

The main purpose of the study is to identify the types of errors committed by secondary school students in graphical solution to inequality problems. Specific objectives include:

1. Identification of types of errors committed by secondary school students in solving inequality problems using graphs.

2. Determine the type of errors that are committed by most of the students used in study

3. Suggest strategies for improving students understanding in the study of graphical solution to inequality problems.

RESEARCH QUESTION

Three research questions guided the study. They are:

1. What type of errors do students commit in their attempt to solve inequality problems using graphical methods as indicated by the frequency?

2. Which of the error type was committed by most students?

3. What strategies can be employed to reduce the frequency of the errors committed by most of the students using graphical method to solve inequality problems as perceived by the students?

RESEARCH HYPOTHESIS

One null hypothesis was used and tested at 0.05 level of significance. H_o: There is no significant difference between the expected and observed frequency scores of types of errors committed by the students using graphical method in solving inequality problems.

SCOPE OF THE STUDY

The study was conducted in Aguata Education Zone in Anambra State. Senior Secondary School Class Two students were used. Graphical methods of solving inequality problems in both one and two variables were used. The study investigated the ability of the students to choose and read scales, plot ordered points on both the number line and the Cartesian plane. It also identified strategies that can be used to reduce the frequency of errors committed by most of the students in using graphical method to solve inequality problems.

SIGNIFICANCE OF THE STUDY

The result of the study is considered important and beneficial to the following people and institutions. To the teachers, it will enable them be exposed to the likely errors students can make in using graphical method in the solution to inequality problems. This will enable the teachers employ likely correct methods that can reduce the frequency of such errors by their students. The result will also help the students acquire the correct cognitive and psychomotor skills that will improve their mastery of inequality problems and hence reduce the frequency of errors associated with the learning of inequality problems using graphical method. The result will also improve the quality of discussion by book writers in respect of this topic. The observed errors and suggested strategies will be incorporated as guide to both teachers and students so as to enhance teaching and understanding and performance by the teachers and students respectively.

METHODOLOGY

The study was the descriptive survey design which used researcher designed five essay type diagnostic questions on graphical solution to inequality problems for data collection on the identification of errors while a 7-item structured questionnaire was used to gather information on the strategies for reducing frequency of the errors in solving inequality problems graphically. The questionnaire items were constructed based on the result from the diagnostic test. The diagnostic test has reliability index of 0.72 established through inter – rater method on a sample of 20 senior secondary school class two students. The questionnaire which was rated based on Likert-type 4-point rating scale of Strongly Agree (SA), Agree (A), Disagree (D) and Strongly Disagree (SD) weighing 4, 3, 2 and 1 respectively, has reliability coefficient of 0.82. Multiple random sampling technique was used to select 92 students from the secondary schools in the zone. The students were administered with the instrument in their

respective schools with the assistance of their mathematics teachers by the end of the week when the topic was taught in the various schools in line with the school scheme of works. The students answer scripts were marked by the researchers with a uniform marking guide prepared by the researchers and validated by an expert in mathematics education and in measurement and evaluation respectively. Types of errors were identified and their frequencies of occurrence tallied. The students were later administered with the questionnaire. Three types of statistics techniques were used. These include common percentages used to answer research questions one and two, Average Weighted Responses (AWR) for research question three and chi- square test for testing the null hypothesis.

AWR = sum of the product of the weight and frequencies divided by the total number of the respondents.

PRESENTATION OF RESULTS

This section presents and discusses the results of the data analysis. Research Question one:What type of errors do students commit in their attempt to solve inequality problems using graphical method as indicated by the frequencies of occurrence table?

Table 1: *Frequencies and proportions of occurrences of the error types committed by the candidates*

S/N	TYPES OF ERRORS	F	PROPORTION
1	Errors in representing given inequality on the number line	68	0.73
2	Inability to indicate and represent correctly the boundary points and lines	59	0.64
3	Errors in identifying the regions that represent solutions to a set of inequalities	`47	0.52
4	Error in representing inequality in one variable on the Cartesian plane	49	0.53
5	Errors in representing inequalities when the coefficient of y variable is negative	75	0.82
6	Errors in locating the points (x, y) in their correct order on the plane	38	0.41
7	Errors in making any of the variables the subjects as may be necessary before plotting the graph	56	0.61
8	Error due to inability to find unknown variables	38	0.41
9	Error due to misplacing of numbers with another in a question	25	0.27
10	Errors in simplification of expression involving brackets and negative signs	42	0.46
11	Errors involved in poor computation and substitution process	25	0.27

Table one above shows the observed types of errors that were committed by the students who were administered with the instrument that required them to attempt problems in inequality using the graph method. The table also shows the proportion of the students who committed each of the observed types of errors.

From the table, errors inrepresenting inequalities when the coefficient of y variable is negative had the highest

frequency (75), followed by errors in representing inequality problems on the number line, inability to identify boundary points and lines (59) was the next type of error, followed by inability to make any of the variables the subject of the formula (56) etc.

Research Question two: Which of the error(s) was committed by most students?

Result from Table 1 clearly revealed that out of 92 students that took the test, 75 students committed errors inrepresenting inequalities when the coefficient of y variable is negative, 68 in inability to represent given inequality on the number line, 59 in inability to indicate and represent correctly the boundary points and lines, 56committed errors in making any of the variables the subjects as may be necessary before plotting the graph.

Research Question three: What strategies can be employed by the teachers to reduce frequency ofthe errors committed by most of the students using graphical method to solve inequality problems as perceived by the students?

Table 2: Summary of students' response scores on strategies for reducing frequency of errors in solving inequality problems graphically.

S/N	ITEMS	SA	A	SD	D	AWR	Decision
1	Teachers should administer readiness test to the students to identify ability to plot ordered pairs and read graphs.	42	46	2	2	3.39	Accepted
2	Teachers should ensure that all students have graph books and mathematical sets.	14	42	20	16	2.67	Accepted
3	Graph boards should be used by teachers to demonstrate the lesson in the class.	36	42	6	8	3.15	Accepted
4	Instructional strategies used should be able to take care of individual differences.	42	40	4	6	3.26	Accepted
5	Teachers should attend to individual students having difficulty.	38	46	4	4	3.22	Accepted
6	Related concepts to inequality should be clearly revised before the introduction of the new topic.	24	28	18	22	2.59	Accepted
7	Student should be encouraged and made to work in pairs during class activities.	35	38	8	11	3.10	Accepted

KEY:AWR = Average Weighted Response.

Decision rule: Any item which has its AWR less than 2.50 is rejected while item with AWR equal to or above 2.50 is accepted. From the table above, it can be seen that all the items had Average Weighted Response (AWR) above 2.50. The items were therefore accepted as the strategies that can be employed by the teachers to reduce the frequency ofthe errors committed by students using graphical methodin solving inequality problems.Hypothesis:There is no significant difference between the observed and expected frequencies on the types of errors committed by the students in graphical solution to inequality problems.

Table 3: *summary of Chi – square analysis of the frequencies of occurrence of errors*

Errors	1	2	3	4	5	6	7	8	9	10	11	X^2_{cal}	X^2_{crit}	Decision
Observed	68	59	47	49	75	38	56	38	25	42	25			
												266.54	18.307	Sig.
Expected	27	27	27	27	27	27	27	27	27	27	27			

Decision Rule: Reject the null hypothesis if the calculated Chi-square is greater than the critical value of 18.307 at 0.05 levels of significance and df of 10. Observation: from table 2, the Chi-square calculated of 266.54 is greater than 18.307, the null hypothesis was therefore rejected. The observed frequencies of the occurrence of the types of errors committed by the students differed significantly from the expected frequencies. The frequencies represent actual proportions of the students who committed the various types of errors in attempt to solve inequality problems using the graphical method.

DISCUSSION

This study was carried out to investigate the types of errors committed by students while solving graphical problems involving inequalities. This was premised on the fact that the achievement of the students in solving problems involving algebraand inequalities in particular, has been poor. Table 1 shows the observed errors committed by the students who were administered with the instrument that required them to attempt problems in inequality using the graph method. The table also shows the proportion of the students who committed each of the observed types of errors. It was observed that the students commit eleven (11) outstanding errors while solving inequality problems graphically. The problems include:Inability to represent given inequality on the number line, Inability to indicate and represent correctly the boundary points and lines, Inability to identify regions that represent solutions to a set of inequalities, Inability to represent inequality in one variable on the Cartesian plane, Inability to represent inequalities when the coefficient of y variable is negative, Inability to locate the points (x, y) in their correct order on the plane, Inability to make any of the variables the subjects as may be necessary, etc. the result from this analysis of errors agrees with WAEC chief examiners' report on mathematics which pointed out that one of the areas where students committed errors was in inequality problems involving graphs. This is also consistent with [5] findings from their analysis of misconception held by mathematics students in algebraic equations, that students had difficulties in separation and transformation of algebraic terms, balancing and knowledge of algebraic structures. Research question two sought to find out the type of errors committed by most students. Result from Table 1 revealed that the errorscommitted by most of the students are those with observed proportion or percentage frequency up to and greater than 50% in the table. These include the following; Inability to represent inequalities when the coefficient of y variable is negative (82%), inability to represent inequality problems on the number line (73%), inability to indicate and correctly represent the boundary points and lines on the plane (64%), Inability to make any of the variables the subjects as may be necessary (61%), and inability to identify regions that represent solutions to a set of inequalities (52%). Greater proportion of the students commit error in representing inequalities when the coefficient of y variable is negative and

in the representation of inequality on the number line and on the Cartesian plane. This is in conformity with [8] observation that students have difficulty in solving inequalities especially in using and applying the inequality rules such as, changing of inequality sign when multiplied or divided by a negative number. The author also observed that pre-service teachers incorrectly represent the solutions of inequalities on the number line.This may be associated with poor skill in reading of scales and ordered pairs. From Table 2, the respondents were of the view that types of errors committed by the students in their attempts to solve inequality problems can be reduced by, administering readiness test to the students to establish the level acquisition of necessary knowledge for the topic. Teachers should also make use of graph boards to illustrate this concept clearly with the students. Students should also be encouraged to work in groups.The respondents are also of the view that teachers give adequate individual attention students as they teach with the aim of reducing frequency of such observed errors in the learning of inequalities.

CONCLUSION

The study investigated the types of errors students commit while solving inequality problems using graphical method. The findings revealed that the type of errors committed by most of the students is errors in representing inequalities when the coefficient of y variable is negative. Followed by inability to represent inequality on the number line. The next type of error that was committed by most of the students was inability to represent the boundary points and lines. The least type of error committed by the students used in the study is errors involved in poor computation and substitution process.

RECOMMENDATION

Based on the findings and observations, the following recommendations were passionately made;

1. In teaching inequality use of scales should be clearly taught
2. Plotting of points involving the ordered pairs should be clearly illustrated using both graph sheets and boards.
3. The idea of subject formula and effect of division or multiplication by negative numbers should be clearly illustrated with the students.
4. Teachers should as a matter of instructional effectiveness, administer a readiness test on thestudents to identify likely difficulties they will encounter while learning graphicalsolution to inequality problems and take care of those difficulties before going into the topic.

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