

Element of Mathematical methods of Physics and Engineering

By

Emmanuel I. Ugwu (Ph.D)

**Industrial physics Department Ebonyi State
University, Abakaliki.**

PREFACE

Mathematical physics is an inter disciplinary field of academic study in between Mathematics and Physics, aimed at studying and solving problems inspired by physics and engineering within Mathematical frame work. It is slightly different from theoretical physics as mathematical physics involves and emphasizes the mathematical rigor in physics just as it can be seen in mathematics while theoretical physics emphasizes on the links to actual observations and experimental physics as may be presented intuitively and heuristically often with approximation by the physicists or engineers.

Generally, for physicists and Engineers mathematics is not only a tool by means of which phenomena can be calculated, but rather it is also the main source of concepts, and principles by means of which new theories can be created from mathematics in the physical sciences and engineering.

One of the problems with courses in mathematical physics is that students do not always see the tie of mathematics with physics and engineering as a result of their poor background in mathematics coupled with their mathematical phobia occasioned by poor background of the subject right from the lower level of their educational carrier.

In this book, we present those concept and mathematical topics that are necessary for any student studying physics and engineering at undergraduate and to some extent at master's level.

The book provides the mathematical topics that are necessary in physics and engineering with techniques for solving them along with many examples.

It is the outgrowth of the lecture notes which I have used in teaching the course over nineteen years stays at Ebonyi State University for mathematical and theoretical physics courses that has been the major courses I have taught at Pg and undergraduate levels. My special thanks go to my colleagues in the department.

I am especially thankful to the members of my family for their understanding and support during the preparation of this book.

Dr. E. I. Ugwu.

SEPTEMBER, 2020.

BIOGRAPHY

Emmanuel Ifeanyi Ugwu(Ph.D) is a lecturer in Ebonyi State University in the Department of Industrial Physics since 1999 till date. My B.Sc. and Master in Science were obtained from Nnamdi Azikiwe University, Awka, Nigeria where I majored in Solid State Physics were in 1994 and 1997 respectively. My Ph.D which I completed in 2014, before then I have been teaching Method of Theoretical/Mathematical Physics in at both undergraduate and graduate levels since I joined university service as a lecturer 1999 and I had published a book titled Fundamental of Mathematical Physics and Engineering, Element of Statistical Physics,etc. My area is specialization Experimental/Computational nanoscience. I have attended and presented papers in so many conferences within and outside and within Nigeria and currently I have more than eighty journal publications and Textbooks coupled with some chapters- contribution to textbooks in some reputable publishing houses. My research interest is in Nanoscience and Electromagnetic wave. which ignited as result of my work from my M.Sc. where I had worked on the growth and characterization of FeS_2 , $FeCl_2$ and many other thin films which eventually ushered me into the world of Nanoscience. After that my passion and research inclination has been on nanoscience. had worked on growth, optical and structural characterization of chalcognide thin film like Copper Sulphide, CdS, MnS, and Antimony Selenide etc I have supervised students on the same areas at both undergraduate and graduate levels. In resent time, we have used a mathematical model in form of beam propagation method to assess the behaviour of electromagnetic wave propagation through thin film medium with consideration given to influence of dielectric constant to the propagating wave applying Green's function and Lippmann-Schwinger equations. My has also done work on Oxide based thin films such as SO_2 , Nikel Oxide, Titanium Oxide, ZnO thin films and other perovskite -Type oxides etc. with a view of ascertaining their potentiality in solar cell and optoelectronic devices to enable us assess which one that is more efficient for commercial application.



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