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Numerical Simulation of a Computer Virus Transmission Model using Euler Predictor Corrector Method

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ABSTRACT

In this paper, a system of nonlinear ordinary differential equations describing the transmission of computer virus in a network was solved numerically using the Euler Predictor Corrector method. To demonstrate the efficiency and accuracy of the proposed method, numerical comparisons are carried out with the results obtained by Runge–Kutta–Fehlberg method. Thus,establishing that Euler Predictor Corrector is a promising method to solve mathematical models on the spread of computer virus.

Keywords: Euler Predictor -Corrector method, Numerical, TransmissionSimulation,Virus

INTRODUCTION

Computer virus is a malicious computer program designed to replicate itself and spread from one computer system to another. Computer virus mainly attack, damage or modify the normal operation of the system files and can be transferred from one computer to another. Hence, it is destructive and contagious. It is spread through the mobile hand disk, an email attachment, download from an infected site, interconnected computers in a network, and use of already infected storage device on unaffected one. These viruses are grouped into file infectors, boot – sector viruses, macro viruses and Trojan horse.

With the advancement in Information and Communication Technology (ICT), communication system, network technology, network information system has become a crucial tool for development of industries, businesses and the countries at large amongst other application areas. The security of the information is one of the important and challenging issues bothering this age of information sharing. Computer virus is one of such threat to information security. Consequently, the attempt to get a better understanding of the computer virus spread dynamics is an important issue for improving the safety and reliability in computer network systems.

There are several similarities between biological virus and computer virus in their mode of propagation. The biological virus can multiply under conducive atmosphere and the infected organism shows its symptoms and can even die in severe cases. So also the computer virus spread from one infected computer to another uninfected one and the infected systems.These similarities between the spread ofcomputer virus and biological virus were first studied by [1, 2]. Piqueira et al [3, 4] studied the epidemiological model of computer viruses. In [5], dynamical models describing the spread of computer viruses were studied and validated using real data.

Mishra et al [6, 7] formulated the mathematical model of internet spread of computer virus. The threshold theory was employed to analyse the propagation of virus and predict the development trend of the computer virus. The analysis of computer virus model was done by [8]. New computers were incorporated into the network and the old ones were removed. Also, the stability conditions of the equilibria are derived. Other works on the model of computer virus are found in [9 -18].

Most models describing the spread of computer virus consists of nonlinear ordinary differential equations with no exact solutions. Thus to study the behavior of such models quantitatively, numerical methods that provide reliable approximate solutions are often used. These methods include Euler's method,Taylor's series method,Picard's method,Trapezoidal methods, etc.

The simplest and straight forward technique available to solve ordinary differential equation is the Euler's method [19]. This method has been used to solve real life problems of involving ordinary differential equations of either linear [19, 20] or nonlinear types [21, 22]. Unfortunately, Euler's method is not reliable for practical computation due to low accuracy and poor stability behavior [19, 20]. To overcome this setback, the Euler Predictor Corrector (EPC) method is presented in [23-25].

Therefore, in this present work, we shall carryout the numerical simulation of an existing computer virus model in [8] and comparison between the proposed method and the famous Runge–Kutta–Fehlberg method will be presented. To the best of our knowledge, this paper gives the first application of EPC method to solve a system of first order nonlinear ordinary differential equation arising from real life problem.

The rest of the paper is organized as follows: Section 2 contains the model assumption and formulation. In section 3, the proposed method is described. Section 4 numerical simulation is presented and lastly, conclusion is contained in section 5.

Model Formulation

A non-linear compartmental model for the spread of computer viruses in a network is considered in [3-5, 8] by stratifying the total population of computers at time t denoted by $N(t)$ into four disjoint compartments which are susceptible compartment $S(t)$ (i.e. groups of computers that likely to be infected) $S(t)$, infected compartment $I(t)$, Removed compartment $R(t)$ either through infection or not and Antidotal compartment $A(t)$(non- infected computers that are equipped with anti-virus program). Thus $N(t) = S(t) + I(t) + R(t) + A(t)$.

The following assumptions were considered to construct the model

1. Only susceptible computers are introduced into the network.
2. The computer population varies with time and is homogenous.
3. Influx rate is not equal to mortality rate.

The model is therefore governed by the following system of non-linear differential equations.

$$\frac{dS}{dt} = Q - (\beta_1 A + \beta_2 I) S - \mu S + \phi R$$
$$\frac{dI}{dt} = (\beta_2 S - \beta_3 A) I - (\mu + \gamma) I$$
$$\frac{dR}{dt} = \gamma I - (\mu + \phi) R$$
$$\frac{dA}{dt} = (\beta_1 S + \beta_3 I) A - \mu A$$

For convenience sake, $S(t), I(t), R(t), A(t)$ and $N(t)$ are written as S, I, R, A and N .

Table 1:Parameters Description and Hypothetical Values

Parameters	Symbols	Estimated Values
Conversion rate of susceptible computers to antidotal ones	β_1	0.00045
Infection rate of susceptible computers	β_2	0.05
Conversion rate of infected computers to antidotal ones	β_3	0.0025
Removal rate for computers	μ	0.05
Influx rate of new computers to the network	Q	1
Recovering rate of removed computers	ϕ	0.8
Removal rate of infected computers	γ	0.96

Note. Source of estimates: [8].

Description of the Proposed Method

Suppose $[0,T]$ be the interval over which we desire to find the solution of a first-order differential equation of the form

$$\frac{dy}{dt} = f(t,y) \quad (2)$$

Subject to the initial condition

$$y(0) = a \quad (3)$$

Then by Euler Predictor Corrector Method, (2) is expressed as

$$y_{0,k+1} = y_k + hf(t_k, y_k) \quad (4)$$

$$y_{j+1,k+1} = y_k + \frac{h}{2}\left(f(t_k, y_k) + f(t_k, y_{j,k+1}) \right)$$

Where the value of y_{k+1} is predicted by (4) and corrected by (5). The iterations for (5) stops when the condition

$$\left| y_{j+1,k+1} - y_{j,k+1} \right| \le tolerance$$

is met. The interval $[0,T]$ is subdivided into k subintervals $[t_k, t_{k+1}]$ of equal step size $h = \frac{T}{N}$.

Thus by Euler Predictor Corrector (EPC), $k = 0, \ldots N-1$ (1) is expressed as

$$S_{0,k+1} = S_k + hf_1(S_k, I_k, R_k, A_k), \qquad I_{0,k+1} = I_k + hf_2(S_k, I_k, A_k),$$

$$R_{0,k+1} = R_k + hf_3(I_k, R_k), \qquad A_{0,k+1} = A_k + hf_4(S_k, I_k, A_k) \tag{6}$$

$$S_{j+1,k+1} = S_k + \frac{h}{2}\left(f_1(S_k, I_k, R_k, A_k) + f_1(S_{j,k+1}, I_{j,k+1}, R_{j,k+1}, A_{j,k+1})\right),$$

$$I_{j+1,k+1} = I_k + \frac{h}{2}\left(f_2(S_k, I_k, A_k) + f_2(S_{j,k+1}, I_{j,k+1}, A_{j,k+1})\right),$$

$$R_{j+1,k+1} = R_k + \frac{h}{2}\left(f_3(I_k, R_k) + f_3(I_{j,k+1}, R_{j,k+1})\right),$$

$$A_{j+1,k+1} = A_k + \frac{h}{2}\left(f_4(S_k, I_k, A_k) + f_4(S_{j,k+1}, I_{j,k+1}, A_{j,k+1})\right) \tag{7}$$

where

$$f_1(S, I, R, A) = Q - (\beta_1 A + \beta_2 I)S - \mu S + \phi R, \; f_3(I, R) = \gamma I - (\mu + \phi)R$$

$$f_2(S, I, A) = (\beta_2 S - \beta_3 A)I - (\mu + \gamma)I, \; f_4(S, I, A) = (\beta_1 S + \beta_3 I)A - \mu A$$

Equation (6) predicts the solution to model (1) while (7) corrects it.

Numerical Simulation

In this section, we validate the accuracy and reliability of the Euler predictor corrector method for step size $h = 0.1$ and $tolerance = \frac{1}{2000}$ with the in- built program for Runge-Kutta-Fehlberg (RKF45) method of step size $h = 0.05$ in maple.

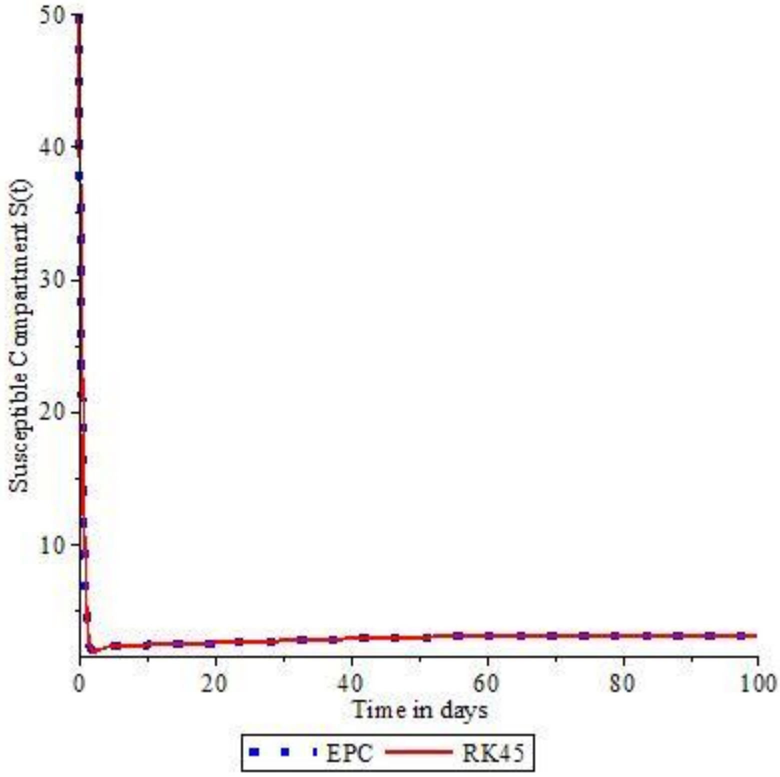

Figure 1: Graphical comparison of $S(t)$

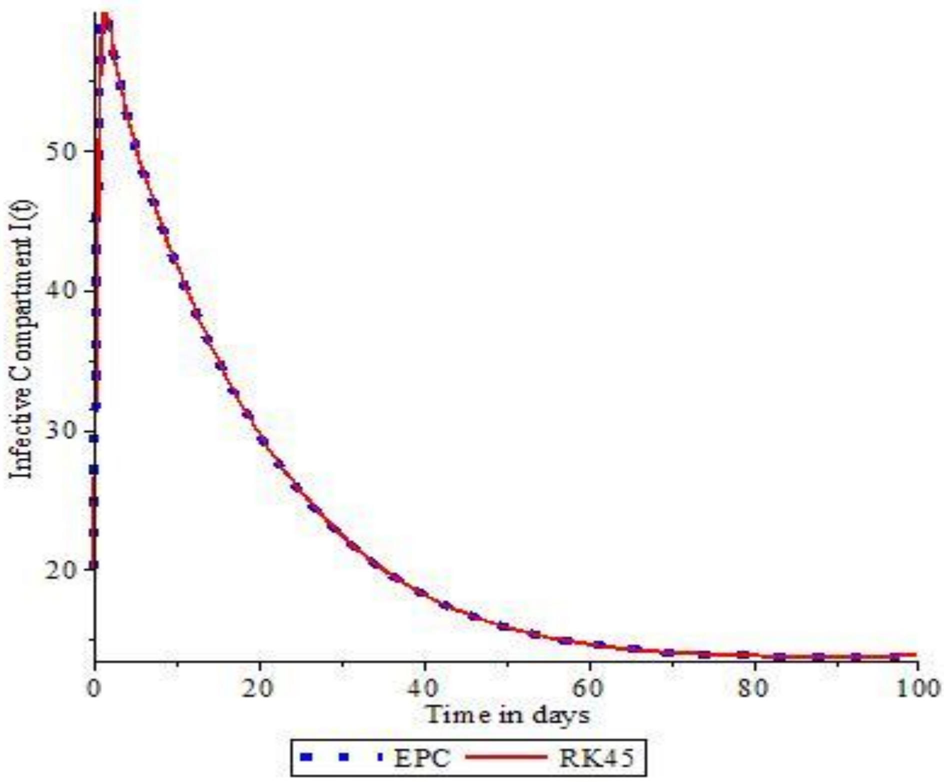

Figure 2: Graphical comparison of $I(t)$

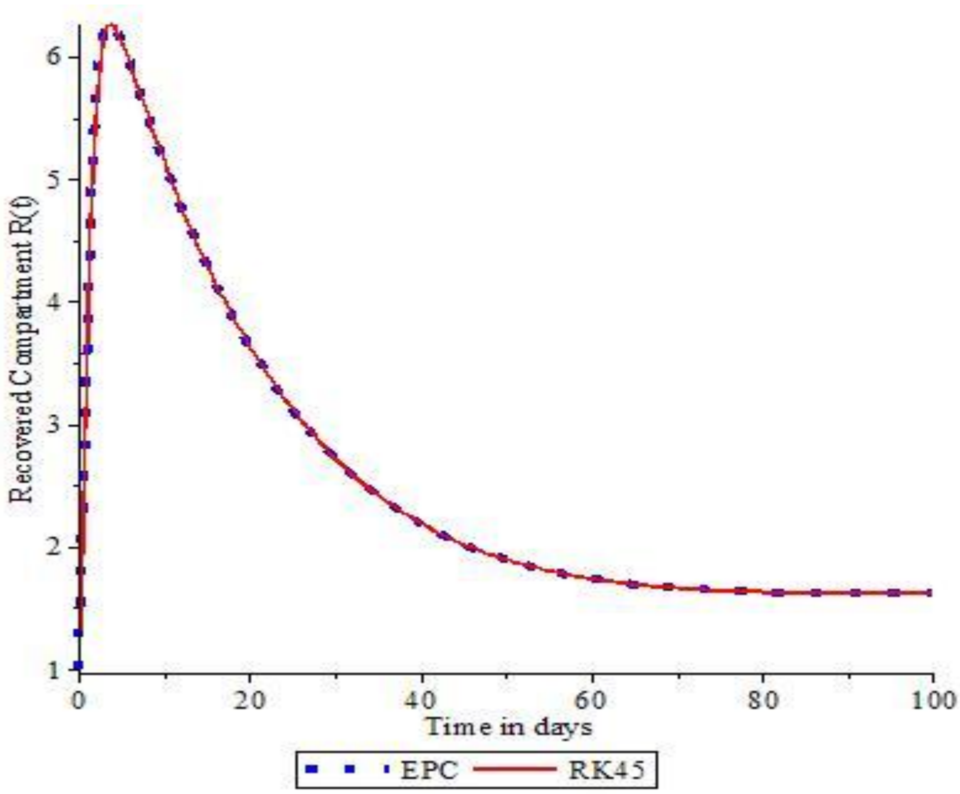

Figure 3: Graphical comparison of $R(t)$

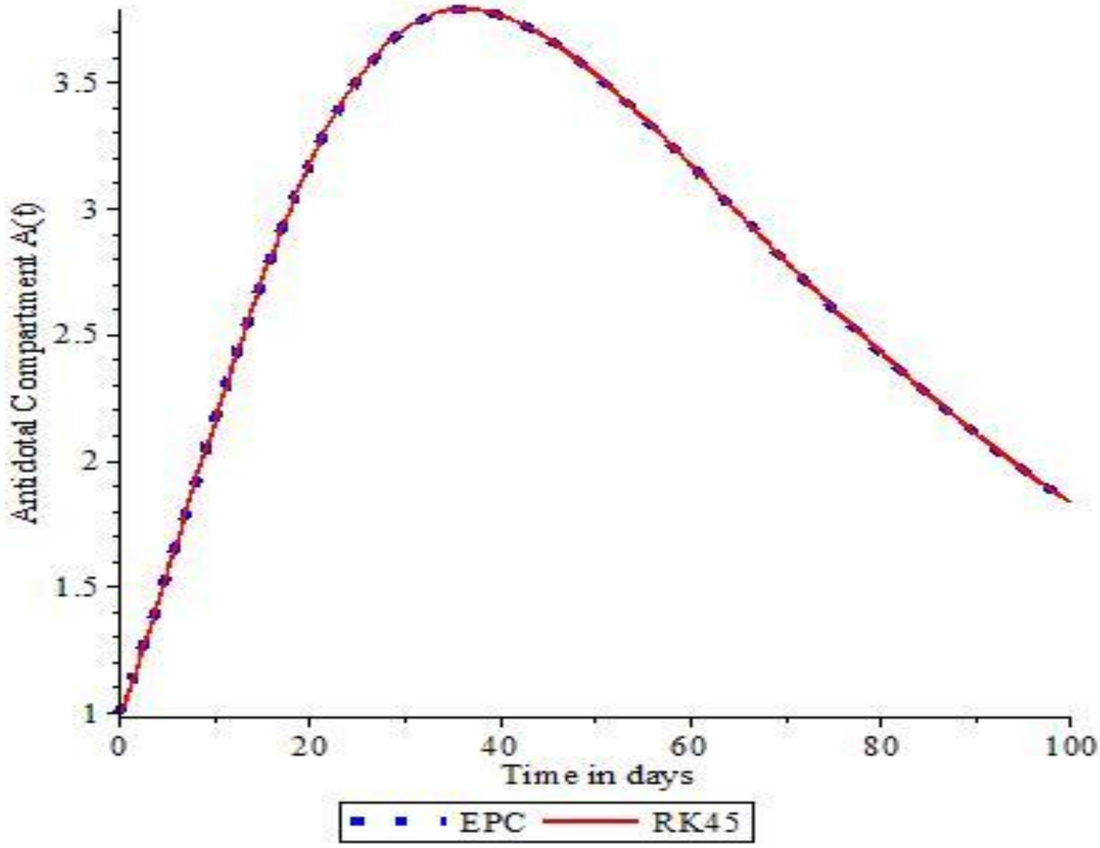

Figure 4: Graphical comparison of $A(t)$

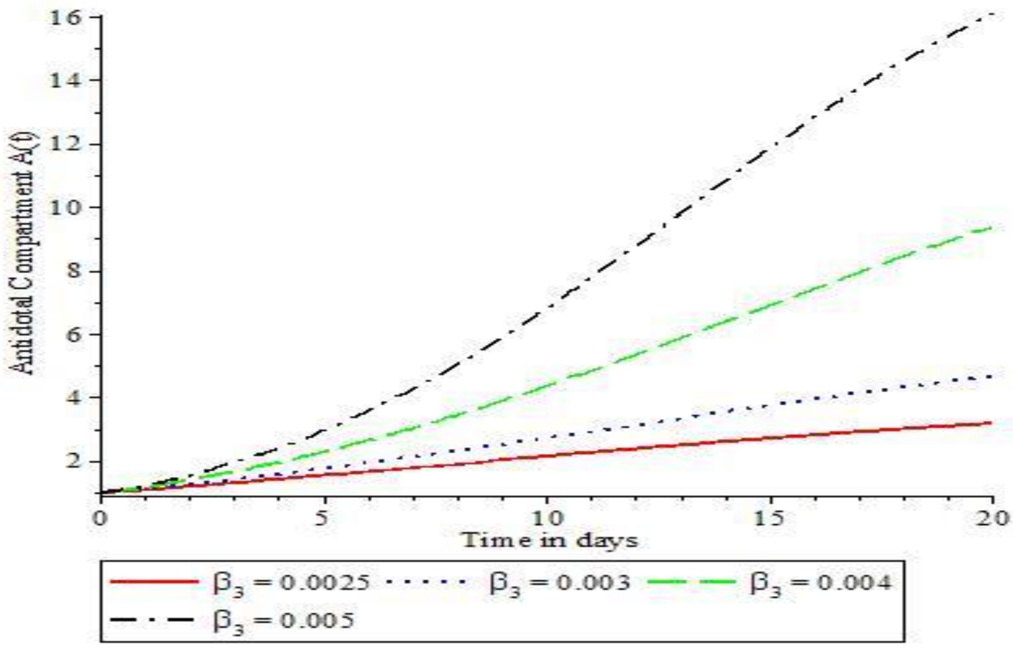

Figure 5: Numerical simulation showing antidotal compartment for various values of β_3

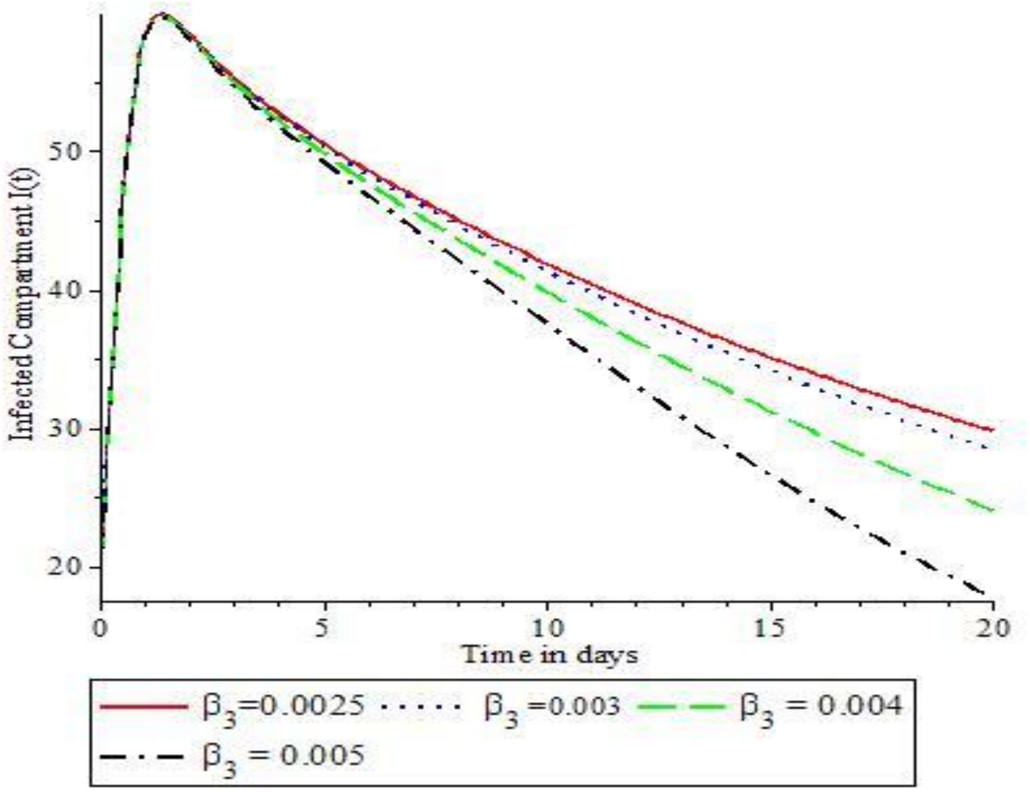

Figure 6: Numerical simulation showing infected compartment for various values of β_3

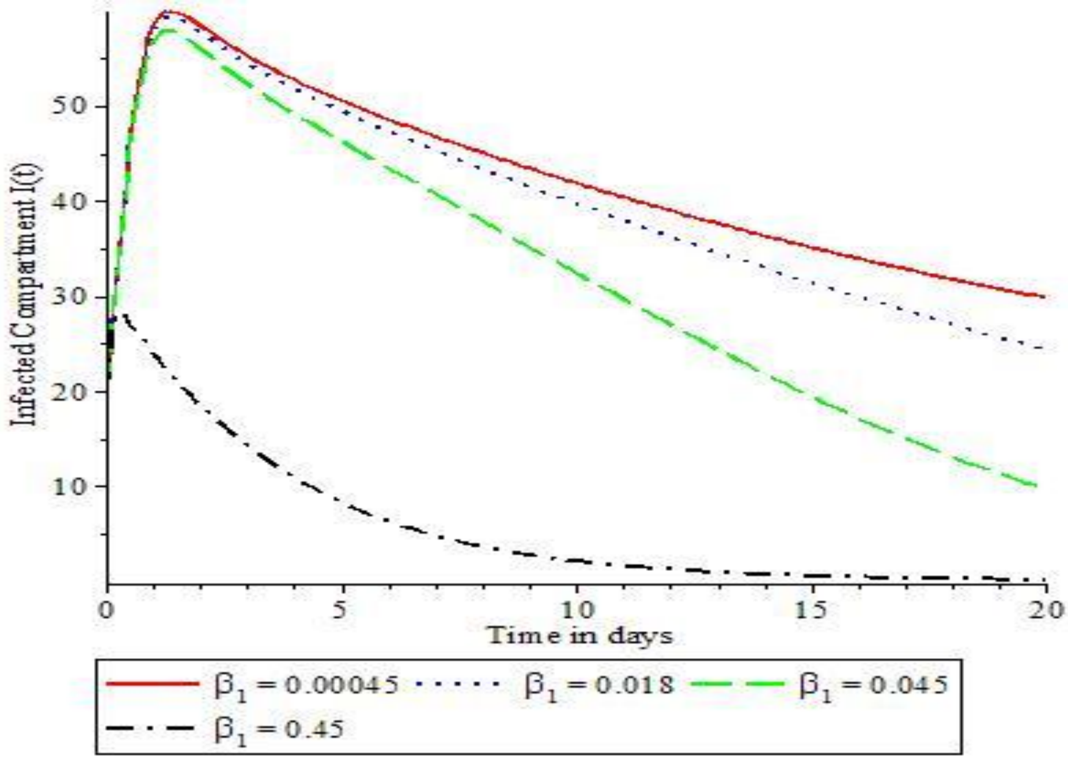

Figure 7: Numerical simulation showing infected compartment for various values of β_1

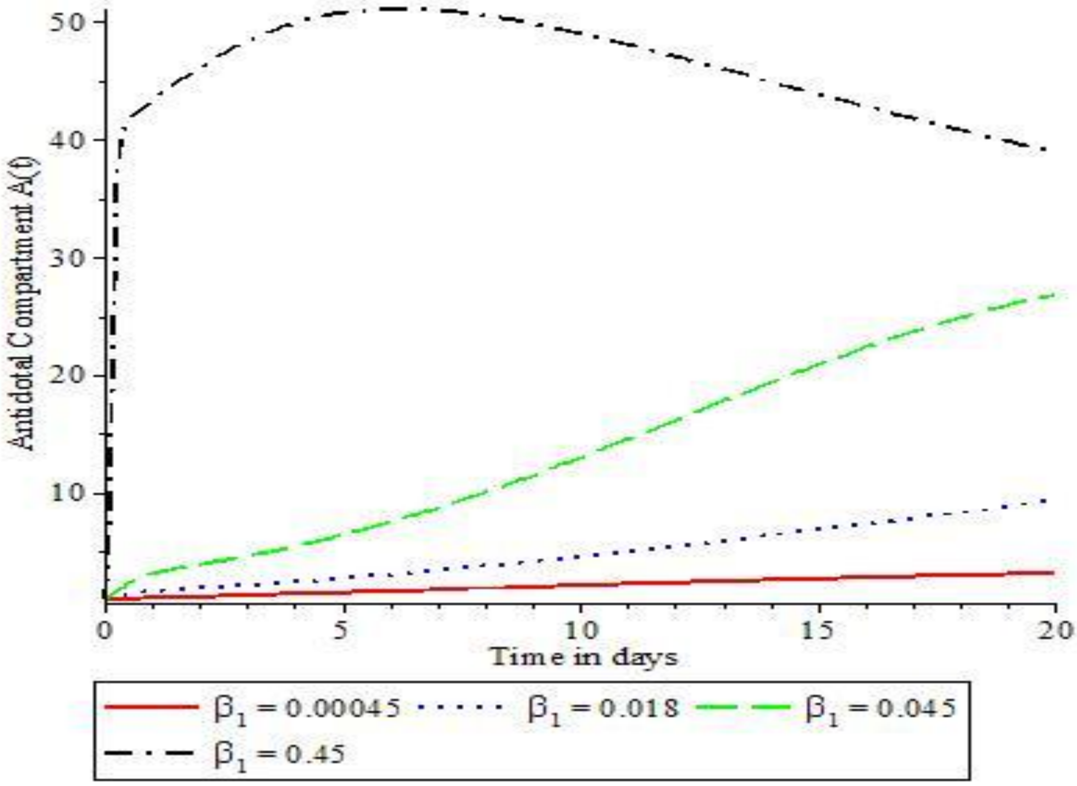

Figure 8: Numerical simulation showing antidotal compartment for various values of β_1

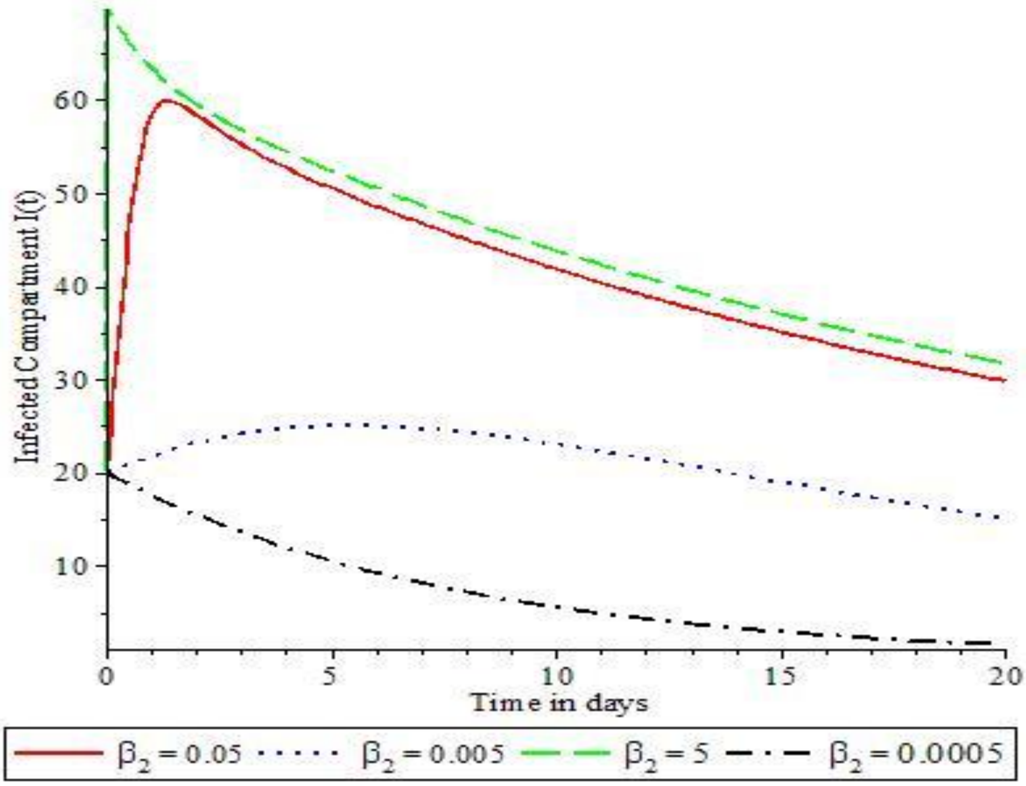

Figure 9: Numerical simulation showing infected compartment for various values of β_2

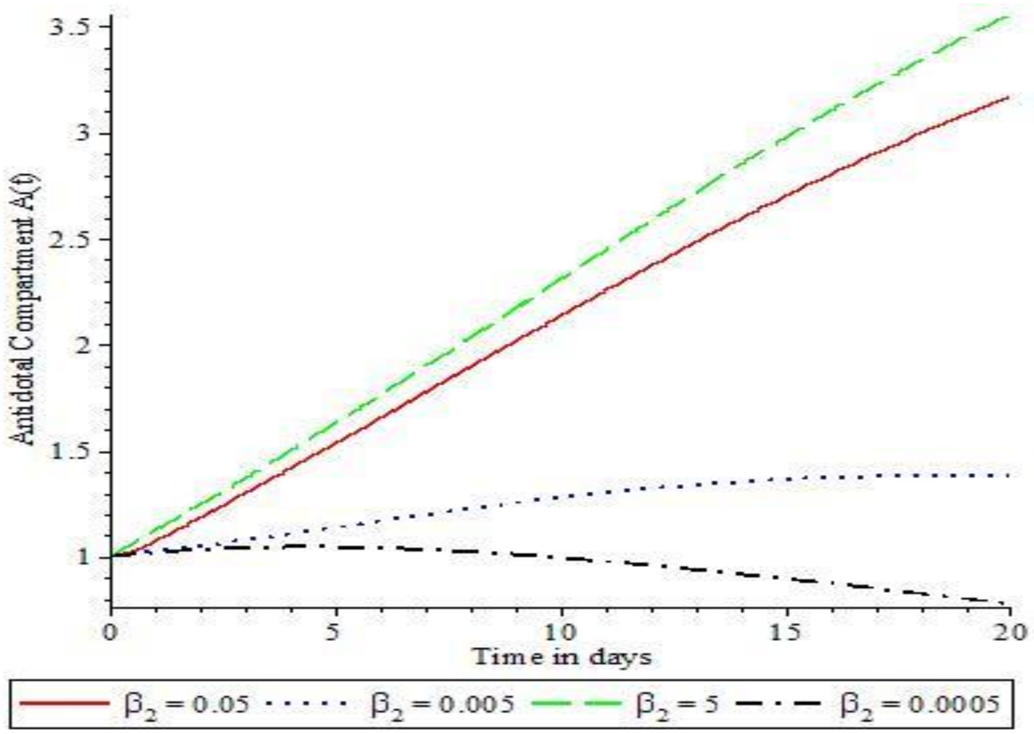

Figure 10: Numerical simulation showing antidotal compartment for various values of β_2

Figures 1-4 shows that the results obtained by Euler Predictor Corrector method is in good agreement with that of Runge-Kutta-Fehlberg methodand correctly produces the dynamics of the model(1) .

Figures 5-10 shows the impact of varying some parameters on the number of computers in infected and antidotal compartments. Increase in the number of antidotal computers as β_3 increases in Fig 5 while increase in $\beta_3$reduces numbers of infected computers in Fig 6.Figures 7and 8 respectively shows that there is reduction in the numberof infected computers and increase in the number of antidotal computers as β_1 increases. The numbers of infected and antidotal computers increases as β_2 increases in Figures 9 and 10 respectively.

In Table 2, we present the absolute differences between ECP solution on step size $h = 0.1$and RKF45 solution on step size $h = 0.05$.

Table 2: Absolute differences were obtained by using RKF45 method of step size $h = 0.05$ and Euler Predictor Corrector for step size $h = 0.1$.

$$\Delta = \left| EPC_{h=0.1} - RKF45_{h=0.05} \right|$$

Time	ΔS	ΔI	ΔR	ΔA
0	0	0	0	0
10	1.36759×10^{-6}	2.57970×10^{-4}	3.17061×10^{-5}	3.21294×10^{-4}
20	8.30662×10^{-6}	3.80113×10^{-4}	4.31844×10^{-5}	4.55059×10^{-4}
30	1.39667×10^{-5}	4.22702×10^{-4}	4.93406×10^{-5}	4.94597×10^{-4}
40	1.88991×10^{-5}	4.03629×10^{-4}	4.78374×10^{-5}	4.62134×10^{-4}
50	2.13624×10^{-5}	3.55487×10^{-4}	4.24772×10^{-5}	3.99006×10^{-4}
60	7.11209×10^{-5}	3.00799×10^{-4}	3.60877×10^{-5}	3.32066×10^{-4}
70	1.91135×10^{-5}	2.50190×10^{-4}	3.00691×10^{-5}	2.72640×10^{-4}
80	1.64292×10^{-5}	2.07073×10^{-4}	2.49014×10^{-5}	2.23526×10^{-4}
90	1.37635×10^{-5}	1.71796×10^{-4}	2.06513×10^{-5}	1.84086×10^{-4}
100	1.14193×10^{-5}	1.43235×10^{-4}	1.72093×10^{-5}	1.52680×10^{-4}

CONCLUSION

A four nonlinear compartmental model describing the transmission dynamics of computer virus in the presence of anti- virus program in a network was considered and solved numerically using the Euler Predictor Corrector method. The proposed method was compared with the RKF45 method to establish its accuracy and flexibility. Based on its excellent agreement with RKF45, Euler Predictor Corrector is therefore a mathematical tool which provides accurate numerical solutions for computer virus transmission models presented by system of nonlinear ordinary differential equations and can be applied to solve other problemsinvolving ordinary differential equations.

Other important findings suggested by numerical simulation include:

1. Increase in conversion rate of susceptible computers to antidotal computers (β_1) leads to an increase in antidotal computers but reduces infected computers.
2. Both infected and antidotal computers increases as the conversion rate of susceptible computers to infected computers(β_2).
3. Antidotal computers increases as conversion rate of infected computers to antidotal computers(β_3) increases but reduces the numbers of infected computers.

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