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A Review of Relevant Processing Techniques for Handling Array Non-Idealities

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ABSTRACT

Sensor arrays signal processing methods are key technologies in many areas of engineering technology applied widely in wireless communication systems, radar and sonar as well as in biomedicals. The demand from the wireless communications market has been the driving force for the development of smart antenna systems, diversity techniques, multiple-input multiple-output (MIMO) systems, and spatial division multiple access (SDMA). In this study, various techniques applied in processing sensor array with non idealities were presented with deeper insight in Auto calibration technique as well as wavefeild modeling and manifold separation approach. Qualitative comparison of the presented techniques was done. Finally, for each approach, appropriate application scenario was identified to assist designer in making choice of technique to employ depending on their specific requirement for array processing.

Keywords: signal processing; array non-idealities; wireless communication; wavefeild modeling; auto caliberation; MIMO-system.

INTRODUCTION

Array processing is an area of study devoted to processing the signals received from an antenna array and extracting information of interest. It has played an important role in widespread applications like radar, sonar, and wireless communications. Numerous adaptive array processing algorithms have been reported in the literature in the last several decades. These algorithms, in a general view, exhibit a trade-off between performance and required computational complexity [1]. In array signal processing we are usually either interested in enhancing, synthesizing, characterizing or attenuating certain aspects of propagating wavefields using *sensor array*. Synthesizing or producing a wavefield refers to generating a propagating wavefield with a desired spatial spectrum in order to focus the transmitted energy towards certain locations in space. Characterizing a propagating wavefield on the other hand means to determine its *spatial spectrum*, so that information regarding the location of the sources generating the wavefield can be obtained with ease. Finally, attenuating or enhancing a received wavefield based on its spatial spectrum refers to the ability of canceling interfering sources or improving the signal-to-interference-plus-noise ratio (SINR) and maximizing the energy received from certain directions. The main factor limiting the performance of high-resolution and optimal array processing methods as well as in the tightness of related theoretical performance bounds is known to be the accuracy of the employed array steering vector model [2].

For effective array signal processing, some methods have been developed over the years and some relevant of these techniques are studied and presented in this paper. They would include: wavefeild modeling and manifold separation, Array mapping techniques, Auto calibration technique, interpolation of calibration matrix and use of uncertainty set on the steering vector.

High-resolution direction-of-arrival (DoA) estimation requires an accurate array response model, which is usually achieved by measuring the response for given directions of the sources and employing interpolation [3]. an array auto-calibration approach is capable of estimating geometrical (array shape), gain and phase uncertainties associated with an array of sensors. As opposed to other data-based array calibration techniques in the literature, no transmitting sources are required (either pilot sources or sources of opportunity). Instead, elements in the array operate as transceivers which are utilised to auto-calibrate the array. Hence, unlike in the case of pilot calibration approaches, all elements are at unknown locations. Under this scenario, the distance of the transmitting elements from the array is small compared to the aperture of the Rx-array formed by the remaining elements. Hence, the standard plane wave propagation assumption used in array processing is no longer valid and a spherical wave propagation model should be considered[4]. In this work, Wavefield Modeling array processing technique is reviewed elaborately establishing the link between the technique and other relevant techniques discussed. The general architecture of Wavefeild modeling and manifold separation technique is discussed.

Array Signal Processing overveiw

A sensor array is a collection of sensors located at distinct spatial locations used to sample signals in space. A wavefront which propagates across the array of sensors is recorded by each sensor and the observed multichannel output is known as array signal. Depending on the sensors used, e.g. antenna, microphone, or hydrophone, sensor arrays can be employed in different research areas such as radio frequency scenarios, biomedical studies, acoustic or under-water environments and automation pollution control for instance, With the help of mobile carriers sensor array, pollutant concentration measuring devices are placed in regions with highest concentration of pollutants. Hence, optimally protecting the environments [17].

Generally, the sensors' outputs contain information of a signal waveform which is corrupted by noise and other interferences constituting the nonidealities. The array outputs also contain information on the sensor array such as its geometry, elements characteristic and structural imperfections. Array processing consists of using the multichannel observations collected by a sensor array in an optimal manner in order to detect signals or estimate their parameters. Adaptive beamforming, interference cancellation, and high-resolution direction finding are some of the important areas of sensors array processing [5]. The development of the signal model is based on a number of simplifying assumptions [6]. Throughout this work the sources are assumed to be narrowband, where narrowband means that the signal bandwidth is small compared to the inverse of the propagation time of the wavefront across the antenna array aperture. The physical size of an antenna array, measured in wavelength, is known as the array aperture whereas the effective aperture is the array aperture seen from a certain direction [7].The sources are assumed to be situated in the far field of the array and considered as concentrated entities (point emitters). In addition, it is assumed that the propagation medium is homogeneous, i.e. not dispersive. Consequently, the waves impinging at the sensor array can be considered to be planar [8]. Under these assumptions the sensors observe time delayed versions of the same signal and the location of the emitter may be characterized by the direction-of-arrival (DoA) of the transmitted wavefront [6]. The angular information of the impinging wavefront is contained in the array response, also known as array steering vector $a(\varphi, \theta) \in C^{N\times 1}$. The collection of the steering vectors over the parameter space of interest is known as the array manifold.

Real-World Arrays

The model of the steering vector for real-world antenna arrays as expected is more complex than the case with ideal antenna array where sensors are uniformly distributed in space. In literature several models have been proposed. They consider various kinds of non-idealities such as mutual coupling between sensors [11], manufacturing errors related to elements orientation and position [9], and sensor gains [10]. In this thesis all array imperfections are considered jointly. The joint information about imperfections is acquired by array calibration. Depending on the application a sensor array is composed of N arbitrarily placed sensors forming a 1-D, 2-D, or 3-D structure. Conformal arrays [12], biomedical electro-arrays, and sensor networks are examples of such arrays. In practice each sensor has its individual directional characteristics and a phase center which may not correspond to its nominal location. All the above mentioned properties of a real world array constitutes the non-idealities that must be put into consideration when processing these signals.

In practical array processing applications, where DoA estimation or spatial filtering is the goal, employing ideal array models that do not take into account the various impairments discussed above typically leads to a performance degradation [14]. In fact, the limiting factor in the performance of high-resolution and optimal array processing algorithms is known to be the accuracy of the employed array model rather than measurement noise [13].

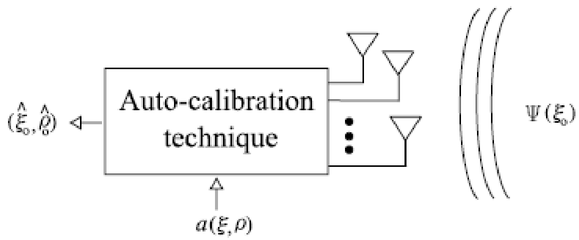
Techniques Used In Handling These Arrays With Non-Idealities:

Auto-calibration techniques

Auto-calibration techniques, also known as self-calibration, amount to estimate wavefield and array parameters simultaneously from a collection of array snapshots. Array parameters, denoted by ρ, may represent unknown array elements' positions, mutual coupling, as well as other array nonidealities. For example, array elements' misplacements in the xy-plane are described by a real-valued vector ρ of dimension 2N. Assume that the array response can be described in a closed-form by both wavefield ξ and array parameters ρ, and denote the corresponding array steering matrix by $A(\xi, \rho)$. The parameter vector ξ may denote the DoAs of the sources, for example. Simultaneous estimation of wavefield and array parameters may be accomplished by the following nonlinear least-squares (NNLS) estimator [18]:

$$\left(\hat{\boldsymbol{\xi}}, \hat{\boldsymbol{\rho}}\right) = \arg\min_{\boldsymbol{\xi},\boldsymbol{\rho}} \operatorname{tr}\left\{\boldsymbol{P}_{\boldsymbol{A}}^{\perp}(\boldsymbol{\xi},\boldsymbol{\rho})\hat{\boldsymbol{R}}_x\right\}, \tag{I}$$

where $P^{\perp}_{A}(\xi, \rho) = I_N - P_A(\xi, \rho)$ denotes an orthogonal projection matrix onto the nullspace of $A^H(\xi, \rho)$. Note that $PA(\xi, \rho) \in C^{N\times N}$ denotes a projection matrix onto the column-space of $A(\xi, \rho)$, and it is given by $P_A(\xi, \rho) = A^{\dagger}(\xi, \rho)A(\xi, \rho)$. When measurement noise is zero mean complex-circular Gaussian distributed, and the unknown parameters are deterministic, (I) is known as deterministic, or conditional, maximum likelihood estimator (MLE) [15].



Typical auto calibration technique (Source [19])

In general, both wavefield and array parameters are not simultaneously identifiable from a collection of array snapshots, unless additional assumptions regarding the array sensors' locations, and the number of sources generating the wavefield are made [16]. For example, the DoAs and the array elements positions are not simultaneously identifiable since the DoA is defined by the relative phases among array elements, which in turn depend on the array sensors' locations. In case the DoAs and array configuration can be determined up to a rotational ambiguity, the spatial signature of the sources may be determined uniquely, which can be used to estimate the transmitted signals [16].

The Bayesian approach combines prior information and observed data in an optimal manner. The effect of the prior distribution diminishes as the number of observations grow. In principle, the Bayesian framework also allows one to "integrate out" the unknown array parameters ρ, and obtain an estimator that is a function of the wavefield parameters ξ, only [69]. Typically, in array processing applications one is mainly interested in the wavefield parameters, and ρ are commonly regarded as nuisance parameters. However, such a task of "integrating out" the array nonidealities is typically extremely challenging due to the non-trivial parameterization of ρ in the array response. For example, in case the parameter vector ρ denotes array elements' misplacements, which are assumed to obey a truncated Gaussian distribution, the pdf of the multivariate observations after marginalizing ρ may not be found in a closed-form.

An alternative approach consists in employing subspace-based estimators for the wavefield parameters ξ that take into account the second order statistics of the array parameters by properly weighting the signal or noise subspaces of the sample covariance matrix.

Bayesian-type of estimators are sometimes regarded as being robust to uncertainties in the array response [9]. In particular, the variation of the array response is taken into account in terms of a known prior distribution for the array nonidealities. However, obtaining a prior distribution describing accurately the array nonidealities may be extremely challenging in practice, and Bayesian-type of estimators may be sensitive to misspecifications of the employed distribution.

Uncertainty sets on the array steering vector

The use of uncertainty sets is understood as a minimax approach where the goal is to optimize a given performance criterion under a worst-case scenario. Robust array processing methods acknowledge that the underlying assumptions regarding the propagating wavefield, sensor array response, and noise statistics may not hold. They trade-off optimality for reliable performance under such circumstances. One such an approach amounts at optimizing certain performance criteria under the assumption that the uncertainty about the real-world array steering vector can be bounded. Geometrically, this may be understood as having an ellipsoid enclosing the maximum uncertainty regarding the real-world array steering vector[19]. Uncertainty sets on the array steering vector have been mainly employed in the context of minimum variance adaptive beamforming [20].

Array mapping techniques

Array Interpolation Techniques aim at replacing the real-world array response in the acquired array snapshots by the response of an ideal sensor array, known as virtual array [15]. Typically, this is accomplished by linearly transforming the array output data in a manner that the transformed data approximates those acquired by employing a virtual array. Computationally efficient array processing methods developed for ideal ULAs and URAs, such as root-MUSIC, ESPRIT, and spatial smoothing, may be employed with real-world sensor arrays and nonidealities by exploiting the transformed data as well as the structure of the virtual array [14].

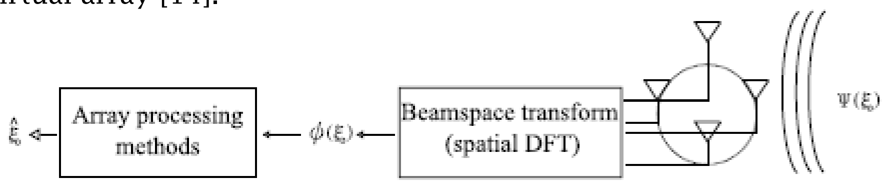


illustration of array processing with spatial DTF (Source [19])

Phase-Mode Excitation is a technique employing a spatial Fourier series representation in order to synthesize arbitrary excitation functions, and consequently far-field patterns, of uniform circular as well as spherical arrays [21]. Originally developed for continuous apertures, the phase-mode excitation technique is also applicable to practical arrays that consist of a few elements. The phase-mode excitation technique is also closely related to the wavefield modeling principle and manifold separation technique.

Extensions of the phase-mode excitation technique, or beamspace transform, to uniform circular arrays in the presence of mutual coupling as well as other perturbations of the array manifold coupled with its extention to spherical arrays can be found in, form the basis to the research topic of *spherical microphone array processing.*

Array calibration measurements

Array calibration aims at acquiring the array response of real-world arrays through measurements from a number of different locations of angles. They capture the combined effects due to mutual coupling, cross polarization effects, and mounting platform reflections, in addition to array elements' misplacements and individual beam-patterns. Typically, array calibration measurements are acquired in controlled environments such as anechoic chambers, and allow for obtaining an accurate and complete description of the radiating characteristics of real-world arrays. High resolution radar, satellite, and communication systems often require such accurate antenna array responses that are typically obtained from array calibration measurements [45].

The most commonly used approach for array calibration measurements acquires the array response to a known active source, called probe, at different angles and polarizations. The antenna array is typically mounted on a mechanical platform, known as positioner, that rotates the sensor array in azimuth ϕ and co-elevation ϑ angles while the probe is held fixed.

Dual-polarized probes are typically employed since one may acquire the array response to horizontal and vertical polarizations without the requirement of rotating the probe as well.

Wavefield Modeling And Manifold Separation

Wavefield modeling is a formalism for array processing where the array output is written as the product of a wavefield independent matrix called array sampling matrix and an array independent vector known as coefficient vector [30]. The sampling matrix depends on the employed sensor array only while the coefficient vector depends on the wavefield. Array sampling matrix and coefficient vector are independent from each other. Manifold separation stems from wavefield modeling and it is a spatial Fourier series of the array manifold and steering vector [9]. The sampling matrix fully describes the employed sensor array, including its geometry and nonidealities, while the coefficient vector uniquely characterizes the received wavefield. Thus, the coefficient vector carries information regarding the wavefield parameters such as DoAs and polarization of the sources in a manner that is independent from the sensor array[19]. Wavefield modeling and manifold separation provide more insight into applications dealing with sensor arrays. For example, such a formalism is appropriate in assessing the fundamental performance limitations of real-world sensor arrays. Moreover, many of the computationally-efficient array processing methods originally developed for ULAs may be employed on sensor arrays with arbitrary geometries and nonidealities.

Wavefield modeling and manifold separation are important results in many fields of engineering employing sensor arrays. The array output is decomposed into two independent parts: the coefficient vector depending on the wavefield only and the sampling matrix depending only on the sensor array, including nonideaities. Describing array nonidealities as well as the array geometry and directional beam patterns of the array elements in a nonparametric manner by the sampling matrix is convenient in array processing. Indeed, such an approach overcomes the challenging task of describing complex electromagnetic interactions among array elements in
a closed-form. These results allow for employing high-resolution and optimal array processing methods in real-world arrays with nonidealities. Array calibration measurements do not limit the practical relevance of wavefield modeling or manifold separation. The sampling matrix may
also be estimated from the array output in multipath channels given that the spatial distribution of the wavefield is known. This is equivalent to know the coefficient vector used in wavefield modeling. Also, the rational behind auto-calibration methods may be employed for relaxing the
requirement of a fully known coefficient vector when estimating the sampling matrix.

The superexponential decay of the sampling matrix is one of the most important properties in wavefield modeling and manifold separation. It is useful in a variety of tasks including array processing, calibration measurements, MIMO systems, and indoor positioning [22].
The superexponential decay is a rigorous argument for considering that a few columns of the sampling matrix describe most of the array characteristics including its nonidealities. Such a result also leads to denoising of calibration measurements and data compression. The physical interpretation of the superexponential decay is that of attenuating the coefficients of the wavefield corresponding to spatial harmonics for increasing orders. It may be understood as the well-known limit in resolution of sensor arrays imposed by their finite aperture.
The equivalence matrix is useful in many areas of engineering dealing with data on a spherical manifold. The ability to write vector spherical harmonics in terms of an exact and finite 2-D Fourier basis expansion as well as the one-to-one relationship among spherical harmonic spectra and 2-D Fourier spectra provides more insight into many of the well-known discrete spherical harmonic transforms. The equivalence matrix proves useful in array processing by enabling a reformulation of wavefield modeling and manifold separation in terms of 2-D Fourier basis. This is important since array processing methods may exploit the computational efficiency and widespread usage of the FFT regardless of the sensor array geometry. Moreover, a rigorous theoretical justification for using the 2-D EADF in any array processing task as well as for improving its estimation variance is obtained by employing the equivalence matrix as well.

Other important results that follow from the equivalence matrix include a novel fast vector spherical harmonic transform and sampling theorems for exact reconstruction on the 2-sphere.

Wavefeild Relationship to local interpolation of the array calibration matrix

Prior knowledge on the location of the sources generating the received wavefield may be incorporated to the array sampling matrix [23]. This is useful when the sources are known to be confined to an angular sector C. Wavefield modeling and manifold separation are then valid over
C, only. The basis functions employed in manifold separation and in obtaining the coefficient vector ψ(k) in wavefield modeling are then orthogonal on the angular sector C and not in whole angular domain S1. This prevents from using Fourier basis since such basis functions are not, in
general, orthogonal on C. Orthogonal basis on an angular sector C may be obtained from Fourier basis by means of Gram-Schmidt orthogonalization, for example [23]. One may still consider
using local basis for decomposing the array steering vector in a piece-wise manner, rather than global basis. Typically, such an approach increases the computational complexity of array processing techniques and may reduce the convergence rate of gradient-based optimization
methods. For example, using the root-MUSIC technique requires finding the roots of n different polynomials while the maximum step-size of gradient-based methods is limited by the size of each angular sector C.
Wavefield modeling and manifold separation using global basis, with a single array sampling matrix, are thus generally preferred when a sensor array is to be deployed on an environment where the sources may span the whole angular region.

	Techniques					
Features	US	ACM	LI	AIT	BT	WM/MS
Applicable to arbitrary array geometries	+ +	+ +	+ +	+ +	- -	+ +
Robustness to modeling errors	+ +	+	- -	- -	-	- -
Characterization of real-world array responses	- -	- -	+	-	- -	+ +
Incorporation of prior knowledge about sources' locations	+ +	+	+ +	+ +	- -	+ +
Computational complexity	- -	- -	-	+	+ +	+
Requirement for user-design parameters	-	- -	- -	- -	+ +	+ +

Table 1.0 Qualitative comparison of array processing techniques for dealing with non-idealities (source [19])

US => Uncertainty Set on the steering Vector
ACM=> Array Calibration Measurement
LI=> Local Interpolation
AIT=> Array Interpolation Technique
BT=> BeamSpace Transform

WM/MS=> Wavefeild Modeling and Manifold Separation

SUMMARY AND DISCUSSION

Real-world sensor arrays are typically composed of elements with individual directional beam patterns. They are subject to misplacements of the array elements, mutual coupling, cross-polarization effects, and mounting platform reflections. Dealing with such non-idealities is crucial in any array processing application in order to obtain optimal results as well as avoid systematic errors and excess variance. Table 1 includes a qualitative comparison of the array processing techniques considered herein.

Auto-calibration methods (ACM) may be useful when both wavefield parameters and array non-idealities are described in a closed-form. However, real-world directional beam-patterns, cross-polarization effects, and mounting platform reflections are extremely challenging to express in a closed-form. Even if a closed-form expression for the uncalibrated steering vector is acquired, wavefield parameters and array non-idealities may not be simultaneously identifiable. Such a difficulty with parameter identifiability may be relaxed by assuming that the array nonidealities are random quantities. Typically, Bayesian-type of estimators require the second-order statistics of the array nonidealities to be fully known, which may not be practical or realistic.

A more conservative approach to dealing with array nonidealities consists in employing uncertainty sets (US) on the array steering vector. Uncertainty sets do not require a closed-form expression for the array nonidealities and may use array calibration measurements. However, uncertainty sets trade-off optimality for robustness. Uncertainty sets may also require user-design parameters for bounding the maximal uncertainty, and the computational burden of such an approach may be prohibitive in many array processing applications.

The beamspace transform (BT) is a rather simple and convenient technique that is amenable to many computationally-efficient array processing methods. However, it requires circular or spherical arrays and its performance is typically limited by the number of array elements.

Array interpolation techniques (AIT) are versatile since the practitioner has the freedom to employ most of the computationally-efficient array processing methods originally developed for ULAs on arrays with arbitrary geometries. However, in cases where the practitioner does not have prior knowledge regarding the location of the sources the performance of array interpolation techniques may be rather poor and the corresponding computational burden may be significant.

Wavefield modeling (WM) and manifold separation (MS) may be seen as an alternative approach of describing the array output. The ability to separate the array parameters, including the array nonidealities, from the wavefield parameters is convenient in many array processing methods. Moreover, the linear relationship describing the array output in terms of sampling matrix and coefficient vector makes wavefield modeling and manifold separation a very attractive technique in signal processing. From the comparison shown in table 1 above, wavefield modeling is seen to be the most qualitative approach to handling non-idealities in array signal processing.

CONCLUSION

This review show the strength and weaknesses of various array processing techniques identifying scenario where each of the discussed technique is most convenient and appropriate. Though, from the comparison shown in table 1 above, wavefield modeling is seen to be the most qualitative approach to handling non-idealities in array signal processing. We still believe that it is important for research to be geared towards developing a more convenient array signal processing technique probably one with hybrid of the characteristic features of the techniques discussed in this review.

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